

67. (a) We use conservation of mechanical energy to find an expression for  $\omega^2$  as a function of the angle  $\theta$  that the chimney makes with the vertical. The potential energy of the chimney is given by  $U = Mgh$ , where  $M$  is its mass and  $h$  is the altitude of its center of mass above the ground. When the chimney makes the angle  $\theta$  with the vertical,  $h = (H/2) \cos \theta$ . Initially the potential energy is  $U_i = Mg(H/2)$  and the kinetic energy is zero. The kinetic energy is  $\frac{1}{2}I\omega^2$  when the chimney makes the angle  $\theta$  with the vertical, where  $I$  is its rotational inertia about its bottom edge. Conservation of energy then leads to

$$MgH/2 = Mg(H/2) \cos \theta + \frac{1}{2}I\omega^2 \implies \omega^2 = (MgH/I)(1 - \cos \theta) .$$

The rotational inertia of the chimney about its base is  $I = MH^2/3$  (found using Table 11-2(e) with the parallel axis theorem). Thus

$$\omega = \sqrt{\frac{3g}{H}(1 - \cos \theta)} .$$

- (b) The radial component of the acceleration of the chimney top is given by  $a_r = H\omega^2$ , so  $a_r = 3g(1 - \cos \theta)$ .
- (c) The tangential component of the acceleration of the chimney top is given by  $a_t = H\alpha$ , where  $\alpha$  is the angular acceleration. We are unable to use Table 11-1 since the acceleration is not uniform. Hence, we differentiate  $\omega^2 = (3g/H)(1 - \cos \theta)$  with respect to time, replacing  $d\omega/dt$  with  $\alpha$ , and  $d\theta/dt$  with  $\omega$ , and obtain

$$\frac{d\omega^2}{dt} = 2\omega\alpha = (3g/H)\omega \sin \theta \implies \alpha = (3g/2H) \sin \theta .$$

Consequently,  $a_t = H\alpha = \frac{3g}{2} \sin \theta$ .

- (d) The angle  $\theta$  at which  $a_t = g$  is the solution to  $\frac{3g}{2} \sin \theta = g$ . Thus,  $\sin \theta = 2/3$  and we obtain  $\theta = 41.8^\circ$ .